the parabola is of the second order as long as the microstrains do not exceed 10^{-5}, and progressively transforms into a higher order parabola (**10**). During this part of the test a low increase in the applied load leads to a sharp increase in strain.

The microstrain measurements show that plastic strain sets in progressively. The concept of a material which remains perfectly elastic up to a certain critical stress, beyond which plastic strain sets in abruptly, must therefore be rejected. Strictly speaking, the 'yield point' does not exist.

The microplasticity curve of Fig. 2 can be described by the following equation:

$$\epsilon_p = C_0(\sigma-\sigma_0)+\frac{C_1C_2(\sigma-\sigma_1)^n}{[1-C_1(\sigma-\sigma_1)][1-2C_1(\sigma-\sigma_1)]} \quad (2)$$

where ϵ_p is the microstrain, σ the applied stress, and C_0, C_1 and C_2 depend on the composition and structural condition of the material. In most cases the exponent n is equal to 2, although it can have other values for certain materials. This form of the law of microplasticity is not that supplied by the structural study of the phenomenon (**8**), but that which fits in best with the experimental data for a large number of materials. The first term in equation (2) exists only if $\sigma \geqslant \sigma_0$, and the second only if $\sigma \geqslant \sigma_1$.

Criterion of yielding

The Maxwell–von Mises criterion is known to be that which describes most accurately the onset of plastic strain in a steel subjected to combined stresses. It has recently been shown (**11**) that this criterion derives directly from Eyring's thermodynamic model (**12**), provided that plastic strain under combined stresses is assumed to result from a simple superimposition of plastic shear strains in planes making an angle of 45° with the directions of the principal stresses, each shear strain depending solely on the shear stress in the corresponding plane.

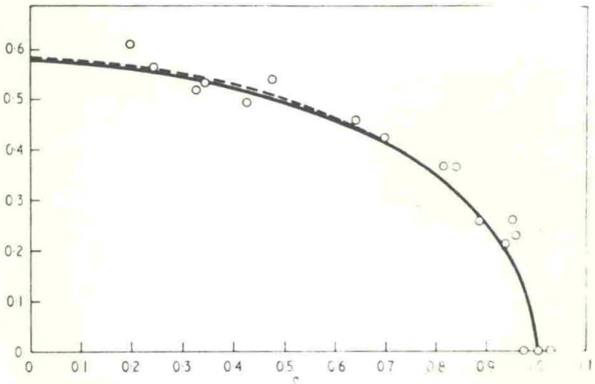
As a mater of fact, if plastic strain is due to a thermally activated movement of dislocations, the shear strain rate, $\dot{\gamma}$, is given by an equation of the type

$$\dot{\gamma} = 2A \exp\left(-\frac{W}{kT}\right)\sinh\left(\frac{V\tau}{kT}\right) \quad . \quad . \quad (3)$$

where k is Boltzmann's constant, T the absolute temperature, τ the shear stress applied to the dislocation, V the activation volume, W the activation energy, and A is a constant which embodies several structural factors. Granting the simple superimposition of plastic shear strains in planes lying at 45° to the principal stresses, the largest strain rate is found in the direction of the highest principal stress, and is equal to

$$\dot{\epsilon}_1 = 2A \exp\left(-\frac{W}{kT}\right)\left[\sinh\frac{V(\sigma_1-\sigma_3)}{2kT}+\sinh\frac{V(\sigma_1-\sigma_2)}{2kT}\right] \quad . \quad . \quad . \quad (4)$$

Assuming that the yield stress corresponds to the stress level at which $\dot{\epsilon}_1$ reaches and exceeds a certain critical threshold value, it is possible to derive the yield stress corresponding to any system of stresses. This calculation has been made in the case of stress systems comprised between pure tension and pure shear stress. The theoretical law thus obtained is shown in Fig. 3, where σ/σ_y has been plotted against τ/σ_y, as is often done for these stresses. This figure also contains the Maxwell–von Mises



— — — Maxwell–von Mises criterion.
——— Law derived from equation (4).
○ Experimental data.

Fig. 3. Plasticity threshold for a mild steel at room temperature according to reference (11)

relation, and a set of experimental points determined at room temperature. It is seen that the theoretical law derived from equation (4) is almost identical with the Maxwell–von Mises relation.

In order to calculate the onset of plastic strain in a steel subjected to combined stresses, it can thus be assumed that the strain results from a simple superimposition of plastic shear strains in planes lying at 45° to the principal stresses: this mode of calculation leads to practically the same results as the Maxwell–von Mises relation.

General relation between stress and plastic strain

The most frequently used strain law is the Lévy–von Mises relation, which assumes that the plastic shear strains in planes lying at 45° to the principal stresses are proportional to the shear stresses in these planes. It is known that stresses and strains can be characterized by two dimensionless parameters, μ and ν which have been proposed by Lode (**13**):

$$\mu = \frac{2\sigma_2-\sigma_1-\sigma_3}{\sigma_1-\sigma_3}; \qquad \nu = \frac{2\Delta\epsilon_2-\Delta\epsilon_1-\Delta\epsilon_3}{\Delta\epsilon_1-\Delta\epsilon_3} \quad (5)$$

and that the use of the Lévy–von Mises relation implies $\nu = \mu$. Since this equality is not confirmed experimentally, it must be concluded that the Lévy–von Mises relation does not give a satisfactory description of plastic strain under combined stresses (Fig. 4).

On the contrary, if it is assumed that plastic shear strains are proportional to the strain rates in the planes lying at 45° to the principal stresses, i.e. that the principal plastic strains can be derived from three equations similar to equation (4), another strain law is obtained. Fig. 4 shows that this law accounts satisfactorily for the experimental results (**11**).

Therefore, the model of plastic strain set out above, which involves the simple superimposition of plastic shear strains in planes lying at 45° to the principal stresses, provides a suitable explanation for both the onset of plastic strain in a material and the law of plastic strain under combined stresses. Hence, this model can replace both the Maxwell–von Mises criterion and the Lévy–von Mises strain law.